Effects of inhomogeneous influence of individuals on an order-disorder transition in opinion dynamics

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We study the effects of inhomogeneous influence of individuals on collective phenomena. We focus analytically on a typical model of the majority rule, applied to the completely connected agents. Two types of individuals A and B with different influence activity are introduced. The individuals A and B are distributed randomly with concentrations ν and $1-\nu$ at the beginning and fixed further on. Our main result is that the location of the order-disorder transition is affected due to the introduction of the inhomogeneous influence. This result highlights the importance of inhomogeneous influence between different types of individuals during the process of opinion updating.

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In recent years, a large class of interdisciplinary problems has been successfully studied with methods of statistical physics, in particular those related to the characterization of the collective social behavior of individuals, such as opinion formation [1,2], the spreading of rumor or disease [3–5], the language dynamics [6], etc. The study of opinion dynamics has become a main stream of research in physics [7–10]. Processes of opinion formation are usually modeled as simple collective dynamics in which the agents update their opinions following local majority [1,7] or imitation [11]. In most of these models, agents are located on the nodes of a graph and endowed with a finite number of available states, e.g., two states—spin up and spin down. Several works have revealed that a given model may exhibit very different (even qualitatively) behaviors depending on its underlying topologies [5]. Recently, Lambiotte has studied the effect of degree dispersity on an order-disorder transition [12].

The heterogeneity of individuals may be an important factor in opinion [13] or other dynamics [14]. In Ref. [14], Szolnoki and Szabó have introduced the effects of inhomogeneous activity of teaching to prisoner's dilemma game. In their model, two types of players that have different teaching activities, which characterizing the master-follower asymmetry between two neighboring players are taken explicitly into account during the strategy adoption mechanism. It was found that the introduction of the inhomogeneous activity of teaching can remarkably enhance the evolution of cooperation [14]. It is natural to consider that the influence between different types of individuals may be different. We think that it will be interesting to introduce different types of people to the opinion dynamics. One can think of a system consisting of two types of people (just like old and young, or attractive and repulsive, individuals in some communities) [14]. In this paper, we consider the opinion dynamics of a system containing two types of individuals (A and B). Our motivation is to explore how the inhomogeneous influence between two types of individuals affects the order-disorder transition. For this purpose, a variant of the majority rule (MR) model introduced by Lambiotte [12] will be considered by assuming only two possible values of ω_{xy}, which characterizing the influence probability between two types of people.

Let us first introduce our opinion dynamics model. The population is composed of N individuals, each of them endowed with an opinion o_i that can be α or β. Two types of individuals ($n_x=A$ or $n_x=B$) are distributed randomly on the nodes of the network. The concentration of individuals A and B are denoted by ν and $(1-\nu)$, respectively. At each time step, one of the individuals is randomly selected. Then one of the following two processes may take place. (i) With probability q, the selected node s randomly changes its opinion,

$$o_s \to \alpha \quad \text{with probability } 1/2,$$

$$o_s \to \beta \quad \text{with probability } 1/2. \tag{1}$$

(ii) With probability $1-q$, two neighboring nodes of s are selected and the three individuals in this triplet update their opinions depending on what types they belong to. First, we define the influence probability between every two neighboring individuals as ω_{xy}. When we update the states of the selected three individuals, if they belong to the same type and there is one individual whose opinion is different from the other two individuals', then the individual whose opinion is in minority changes his opinion with probability 1.0. Thus, we can obtain $\omega_{AA}+\omega_{AA}=1.0$ if the three individuals all belong to the type A, and $\omega_{BB}+\omega_{BB}=1.0$ if they all belong to the type B. So, we define the influence probability between two individuals belonging to the same type as 0.5, i.e., $\omega_{AA}=\omega_{BB}=0.5$. Similarly, we can define the influence probability between two individuals belonging to two different types as $\omega_{AB}=\omega_{BA}=\omega$. For the sake of simplicity, we assume that the influences between individuals in the same type are always stronger or equal to that in two different types, i.e., $0\leqslant\omega\leqslant0.5$. Thus, if this three-individual triplet consists of two types of people, for example, two A and one B, then the individual x whose opinion is in minority changes his opinion with probability 2ω if $n_x=B$ and with probability

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$(0.5+\omega)$ if $n_x=A$. It is straightforward to consider the case of two B and one A. Evidently, if $\omega=0.5$ this model is equivalent to a homogeneous system studied in Ref. [12].

The parameter q involved in the model measures the competition between individual choices and neighboring interactions, i.e., the larger the q is, the more random the system would be; on the contrary, for smaller q, the opinion of individuals would become more homogeneous. In the case $q=0$ and $\omega=0.5$, it is well known that the system asymptotically reaches global consensus where all nodes share the same opinion [7]. In the other limiting case $q=1$, the opinions of individuals are purely random and the average (over the realizations of the random process) number of individuals with opinion α at time t, denoted by A_t, goes to $N/2$ for large t. In the following, we will investigate how the inhomogeneous degree ν of the system and the influence probability ω between different types of individuals affect the order-disorder transition.

In the present model, we assume that the network of individuals is highly connected and homogeneous, i.e., each individual links to all other members in the network. In that case, the mean-filed rate equation for A_t reads

$$A_{t+1} = A_t + q\left(\tfrac{1}{2} - a\right) + (1-q)W, \tag{2}$$

where $a_t = A_t/N$ is the average proportion of nodes with opinion α, and W is the total contribution to the evolution of A_t due to neighboring interactions. The term proportional to q accounts for the random flips and the last term for local majorities. In Eq. (2), the total contribution W is

$$W = W_1 + W_2 + W_3 + W_4, \tag{3}$$

where the four terms are the contribution to the evolution of A_t for the cases of three nodes A, three nodes B, two nodes A and one node B, and one node A and two nodes B, respectively. The probability for two nodes $\alpha(\beta)$ and one node $\beta(\alpha)$ to be selected is $3a^2(1-a)$ $[3a(1-a)^2]$, so that

$$W_1 = \nu^3[3a^2(1-a) - 3a(1-a)^2] = \nu^3[-3a(1-3a+2a^2)], \tag{4}$$

$$\begin{aligned} W_2 &= (1-\nu)^3[3a^2(1-a) - 3a(1-a)^2] \\ &= (1-\nu)^3[-3a(1-3a+2a^2)], \end{aligned} \tag{5}$$

$$\begin{aligned} W_3 &= 3\nu^2(1-\nu)\Big[a^2(1-a)(2\omega) + 2a^2(1-a)\left(\tfrac{1}{2}+\omega\right) \\ &\quad - a(1-a)^2(2\omega) - 2a(1-a)^2\left(\tfrac{1}{2}+\omega\right)\Big] \\ &= 3\nu^2(1-\nu)(4\omega+1)[-a(1-3a+2a^2)], \end{aligned} \tag{6}$$

$$\begin{aligned} W_4 &= 3\nu(1-\nu)^2\Big[a^2(1-a)(2\omega) + 2a^2(1-a)\left(\tfrac{1}{2}+\omega\right) \\ &\quad - a(1-a)^2(2\omega) - 2a(1-a)^2\left(\tfrac{1}{2}+\omega\right)\Big] \\ &= 3\nu(1-\nu)^2(4\omega+1)[-a(1-3a+2a^2)]. \end{aligned} \tag{7}$$

From Eqs. (3)–(7), we obtain

$$W = -3a(1-3a+2a^2)[2\nu^2 - 2\nu + 1 + 4\omega(\nu - \nu^2)]. \tag{8}$$

So the evolution equation for A_t can be written as

$$\begin{aligned} A_{t+1} &= A_t + q\left(\tfrac{1}{2} - a\right) + (1-q) \\ &\quad \times\{-3a(1-3a+2a^2)[2\nu^2 - 2\nu + 1 + 4\omega(\nu-\nu^2)]\}. \end{aligned} \tag{9}$$

It is straightforward to show that $a=1/2$ is always a stationary solution of Eq. (9) due to the existence of symmetry. This is evident after rewriting Eq. (9) for the quantities $\Delta = A - N/2$ and $\delta = a - 1/2$,

$$\begin{aligned} \Delta_{t+1} &= \Delta_t + \frac{\delta}{2}\{3 + 3(2-4\omega)(\nu^2-\nu) \\ &\quad - q[5 + 3(2-4\omega)(\nu^2-\nu)] \\ &\quad - 12(1-q)[(2-4\omega)(\nu^2-\nu)+1]\delta^2\}, \end{aligned} \tag{10}$$

from which one finds that the symmetric solution $a=1/2$ ceases to be stable when $q < \frac{3+3(2-4\omega)(\nu^2-\nu)}{5+3(2-4\omega)(\nu^2-\nu)}$, and that the system reaches the following asymmetric solutions in that case:

$$\begin{aligned} a_- &= \frac{1}{2} \\ &\quad - \sqrt{\frac{3+3(2-4\omega)(\nu^2-\nu) - q[5+3(2-4\omega)(\nu^2-\nu)]}{12(1-q)[(2-4\omega)(\nu^2-\nu)+1]}}, \\ a_+ &= \frac{1}{2} \\ &\quad + \sqrt{\frac{3+3(2-4\omega)(\nu^2-\nu) - q[5+3(2-4\omega)(\nu^2-\nu)]}{12(1-q)[(2-4\omega)(\nu^2-\nu)+1]}}. \end{aligned} \tag{11}$$

The system therefore undergoes an order-disorder transition at

$$q_C(\omega,\nu) = \frac{3+3(2-4\omega)(\nu^2-\nu)}{5+3(2-4\omega)(\nu^2-\nu)}. \tag{12}$$

Below this value, a collective opinion has emerged because of the imitation between neighboring nodes. Let us stress that when $\omega=0.5$ or $\nu=0$ $(\nu=1)$, Eqs. (11), respectively, converge to $a_-=0$ and $a_+=1$ in the limit $q\to 0$. We recover the result $q_C=3/5$ obtained in Ref. [12] in the limiting case $\omega=0.5$. For the homogeneous system ($\nu=0$ or $\nu=1$), one can also recover the known result $q_C=3/5$ [12]. We would resort to pictures to elucidate that the critical value q_C varies with ν and ω as shown in Eq. (12).

In Fig. 1, we show that the critical values q_C of the order-disorder transition change with the fraction ν of individuals A for several different values of ω. We can find from this figure that, when $\omega < 0.5$, the value of q_C decreases monotonously until reaching the minimum value at $\nu=0.5$, which indicates that the more inhomogeneous the system is, the smaller the critical value q_C would be. Our first finding is therefore that the location of the order-disorder transition depends in a nontrivial way on the inhomogeneous degree ν of the system.

Figure 2 shows the analytical results for the dependence of the critical value q_C on the influence probability ω with

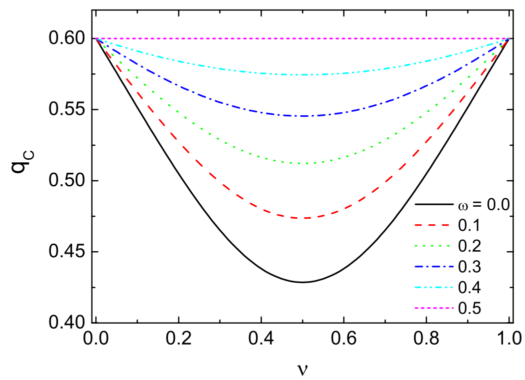

FIG. 1. (Color online) The analytical results of the critical values q_C as a function of ν for different values of influence probability ω (see the plot for detailed values).

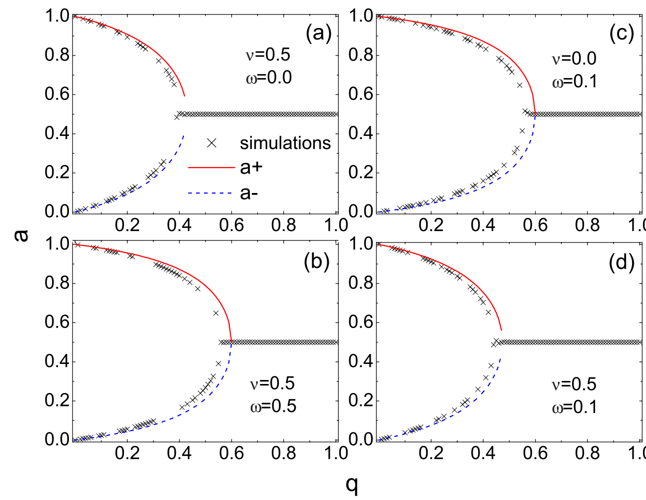

FIG. 3. (Color online) The simulation (crosses) and analytical (colored lines) results of the fraction a of opinion α as a function of q for different values of ω and ν: (a) ν=0.5, ω=0.0; (b) ν=0.5, ω=0.5; (c) ν=0.0, ω=0.1; (d) ν=0.5, ω=0.1.

fixed value of ν. Due to the symmetry of the system for two types of individuals, we only consider the cases for $\nu \leqslant 0.5$. In this figure, q_C increases monotonously with the parameter ω until reaching the maximum value 0.6 at ω=0.5 for each certain value of $\nu > 0$. It indicates that, the larger the influence probability between two types of individuals is, the larger the q_C would be. Our second finding is that the location of the order-disorder transition also depends on the amount of ω: The smaller the ω is, the larger the deviation from the result 0.6 in the homogeneous system would be. The effects of the inhomogeneous influence on collective behaviors may be of some sense in investigating the opinion dynamics in real social systems.

In order to elucidate the behavior of a below q_C, we perform Eqs. (11) from an analytical point of view in Fig. 3. By construction, the random steps of MR are easy to implement in a computer simulation. Simulations were carried out for a population of N=10 000 individuals located on the sites of the completely connected network. Other parameters are denoted in Fig. 3. We study the key quantity of the fraction a of opinion α in the steady state. Initially, the two opinions of α and β are randomly distributed among the individuals with equal probability 1/2. In simulations, we denote α and β by +1 and −1, respectively. Eventually, the system reaches a dynamic equilibrium state. The simulation results were obtained by averaging over the last 10 000 Monte Carlo time steps of the total 100 000. These simulation results are in very good agreement with Eqs. (11) under the critical value q_C, but it appears as a small difference that the value of a is smaller when obtained from simulations than that from analysis near this critical value. This is due to the finite-size effect.

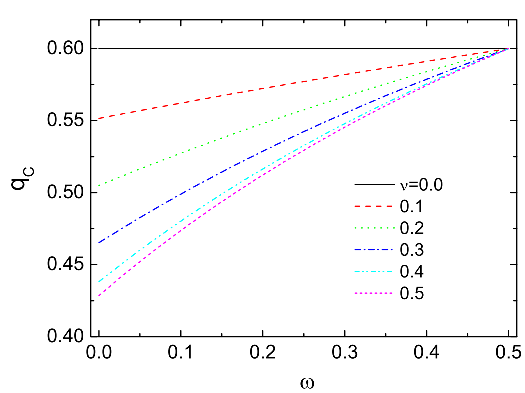

FIG. 2. (Color online) The dependence of the critical values q_C on the influence probability ω between different types of individuals for several different values of ν (see the plot for detailed values).

Finally, we notice that, a transition to a disorder opinion phase was also obtained in Ref. [15]. The differences between our work and Ref. [15] are that, in Ref. [15], Galam considered the contrarians effects on opinion forming, whereas in the present work we focus on the effects of inhomogeneous influence of individuals. The similarities are that in both models, the local majority rule has been used during the process of opinion updating; and the phenomena the order-disorder transitions in opinion dynamics all appeared in both Ref. [15] and our present work. We want to stress that in our model, only the inhomogeneous influence among individuals (which is a typical character of many real social systems) is considered, and one observes rich dynamical phenomena (no other additional constrain conditions are needed), both the phenomena of the order-disorder transitions and the changes of the location of them. The results obtained in Ref. [15] may be set in parallel with recent "hung elections" as occurred in the 2000 American presidential elections and that of the 2002 German parliamentary elections. Due to the somewhat similarity of the results obtained by both models, our present work provides an alternative way to understand the phenomenon of "hung elections" [15].

In summary, we have studied the effects of inhomogeneous influence of individuals on an order-disorder transition in opinion dynamics. We mainly considered the majority rule by introducing two types of people with different influence activity. It was shown that the location of the order-disorder transition depends on the inhomogeneous degree ν of the system and on the influence probability ω between different types of individuals. In social group, the emergence of order means that there exists a clear cut majority-minority splitting, and in this phase one can observe polarization of opinions; while in the disordered phase, there is no opinion dominating with both state densities equal and no global symmetry breaking. From a social point of view, our results suggest that it is more difficult to realize the ordered state in the real world (the value of q_C is smaller in the inhomogeneous case than that in the homogeneous case), because most real social systems behave like the inhomogeneous case. Thus, the inhomogeneous influence of individuals is a correlated factor in opinion dynamics, which plays an important role in the opinion spreading and formation in real systems.

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